

ANALYSIS OF THE OPTIMAL LAMINATED TARGET MADE UP OF
DISCRETE SET OF MATERIALS

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ABSTRACT

A new class of problems has been analyzed to estimate an optimal structure of laminated targets, fabricated from the specified finite set of homogeneous materials. An approximate description of perforation process is based on the model of radial hole extension. The problem is solved by using the needle-type variational technique. The desired optimization conditions and quantitative/qualitative estimations of optimal targets have been obtained and discussed using specific examples.

INTRODUCTION

The problem of optimizing strength properties of inhomogeneous targets under impact of tapered conical indenter was first considered in the study [1]. The qualitative criteria of optimal target structure, developed in this and the following studies (for example [2]), were based on Pontryagin maximum principle [3]. In the previous research an assumption was made about existence of analytical relation between material hardness and density - a class of so called control functions.

In the present investigation the range of control functions belongs to some finite discrete set. This suggests using technique of needle variations [4] when estimating the necessary optimization conditions and constructing computational algorithm.

An approximate analysis of penetration is based on the model of radial hole extension [5].

ANALYSIS

1. Penetration model.

The model of radial hole extension is based on the assumption of radial displacement of material particles under the plane axially-symmetric deformation, caused by penetration of the tapered indenter.

According to [5], the pressure acting on the indenter within the distance $\zeta = x - \xi$ from the front plate surface can be written as

$$p = (1/2)\rho v^2 \left\{ \left(\frac{\partial r}{\partial \xi} \right)^2 \left[\theta(\varepsilon) - \varepsilon / (1 + \varepsilon) \right] + r \frac{\partial^2 r}{\partial \xi^2} \right\} + \\ + (1/2)\rho v \theta(\varepsilon) r \frac{\partial r}{\partial \xi} + (1/2)\sigma_s \left[1 + \theta(\varepsilon) \right] \quad (1.1)$$

where $r = r(\xi)$ is the expression for the generating line of the axisymmetric indenter, ρ is the density, $\theta(\varepsilon) = \ln(1+\varepsilon)$, $\varepsilon = E/[\sigma_s(1+\nu)]$, v is the indenter current velocity, σ_s is the yield stress, ν is Poisson ratio, E is Young's modulus.

The equation of motion for the indenter of mass M has the form

$$(1/2)Md(v^2)/dx = -2\pi \int_0^x \rho(\xi)r(\xi)(\partial r/\partial \xi)d\xi \quad (1.2)$$

with the initial condition $v(0) = v_0$.

2. Optimization problem.

It is convenient to represent the penetration process as a system of differential equations relative to a vector of phase coordinates y with u as a control function

$$dy/dx = f(y, u), \quad y(0) = y_0 \quad (2.1)$$

and to define Freshet's differentiable functionals by

$$F_0[u(\cdot), b] = \int_0^b \rho(x)dx, \quad (2.2)$$

$$F_1[u(\cdot), b] = y^1(b) = 0 \quad (2.3)$$

Insertion of additional phase coordinates and use of (1.1) and (1.2) reduces (2.1) to

$$\begin{aligned} dy^1/dx &= -2(y^1 y^2 + d_1 y^3)/(d_2 + y^4), & dy^2/dx &= y^5 - A(x-h) \cdot h, \\ dy^3/dx &= y^6 - B(x-h) \cdot h, & dy^7/dx &= 2y^7 - C(x-h) \cdot h^2, & dy^8/dx &= A(x) - A(x-h), \\ dy^6/dx &= B(x) - B(x-h), & dy^7/dx &= y^8 - C(x-h) \cdot h, & dy^9/dx &= C(x) - C(x-h), \\ y^1(0) &= v_0^2, & y^i(0) &= 0, & (i &= 2, 3, \dots, 8), \end{aligned} \quad (2.4)$$

where $d_1 = (\text{tg } \alpha)^{-2}$, $d_2 = M d_1^2/\pi$, $y^1 \equiv v^2$,

$$A(x) = 0 \text{ (if } x < 0), \quad \rho(x) \left[\theta(\varepsilon(x)) - \varepsilon(x)/(1+\varepsilon(x)) \right] \text{ (if } x \geq 0),$$

$$B(x) = 0 \text{ (if } x < 0), \quad \sigma_s(x) \left[1 + \theta(\varepsilon(x)) \right] \text{ (if } x \geq 0),$$

$$C(x) = 0 \text{ (if } x < 0), \quad \rho(x) \theta(\varepsilon(x)) \text{ (if } x \geq 0),$$

The size, number and class of materials to be used in the target layers are specified by a distribution of material properties

$$u(x) = \{u_s : x \in [x_s, x_{s+1}), s = \overline{1, n}\}, \quad x_1 = 0, \quad x_{n+1} = b,$$

where n is the number of layers. The value of u_s belongs to a finite set U which corresponds to a given set of materials $u \in U = \{U_1, U_2, \dots, U_q\}$. Here u_s is the material in the s^{th} layer, U_1 is the material number and q is the material quantity.

The stated optimization problem suggests that from all piecewise continuous functions $u(x) \in U$ and numbers $b > 0$ one should choose a control $\{u^0(x), b^0\}$ which will provide minimum for the functional (2.2) under the limiting conditions (2.1), (2.3). The quality criterion may be referred to as a specific plate mass (2.2) subject to $v(b) = 0$ (under the requirement of arrested indenter).

3. Necessary conditions of optimization.

A discrete character of the control function range doesn't allow to generate small variations in the norm $\|\delta u\| = \max_{x \in [0, b]} |\delta u|$. The disturbed control may be written in the form

$$u(x) = \begin{cases} w, & x \leq m, \quad w \in U \\ u^0, & x \notin m, \end{cases} \quad (3.1)$$

where $m \in [0, b^0]$ is the set of measure zero.

An equation for the system (2.1) is expressed in terms of variations and the main terms of functional increments (2.2), (2.3) are given by

$$\begin{aligned} \delta(\bar{y}/dx) - \partial \bar{f} / \partial \bar{y} \delta \bar{y} &= \bar{f}(\bar{y}, u) - \bar{f}(\bar{y}, u^0), \\ \delta F_0 &= \int_m [\rho(w) - \rho(u^0)] dx + \rho[u^0(b^0)] \delta b^0, \end{aligned} \quad (3.2)$$

$$\delta F_1 = \delta y^1(b^0) + \bar{f}[\bar{y}(b^0), u^0] \delta b^0$$

Using the Lagrange identity and desired limiting conditions for the disturbed trajectory one finds an expression for δb^0

$$\delta b^0 = [1/f_1[\bar{y}(b^0), u^0]] \int_0^{b^0} \bar{\psi} [\bar{f}(\bar{y}, u) - \bar{f}(\bar{y}, u^0)] dx \quad (3.3)$$

where the conjugate vector-function $\bar{\psi}$ satisfies

$$d\bar{\psi}/dx = -\partial \bar{f} / \partial \bar{y} \bar{\psi} \quad (3.4)$$

Variation of the minimized functional is written as

$$\delta F_0[u(\cdot), b] = \int_m [H(\bar{y}, \bar{\psi}, u^0) - H(\bar{y}, \bar{\psi}, w)] dx \quad (3.5)$$

In order to make the control function optimal it is necessary to follow the principle of maximum

$$H(\bar{y}, \bar{\psi}, u^0) = \max_{w \in U} H(\bar{y}, \bar{\psi}, w) \quad (3.6)$$

An expression for H is given as

$$H = D[\psi_0(x)A(w) + \psi_0(x)B(w) + \psi_0(x)C(w)] - \rho(w), \quad m \in [b^0 - h, b^0],$$

$$\begin{aligned} H = & \left\{ [\psi_0(x) - \psi_2(x+h)h - \psi_2(x+h)]A(w) + [\psi_0(x) - \psi_0(x+h)h - \psi_3(x+h)]B(w) \dots \right. \\ & \left. + [\psi_0(x) - \psi_0(x+h) - \psi_7(x+h)h - \psi_4(x+h)h^2] \right\} - \rho(w), \quad m \in [0, b^0 - h], \end{aligned} \quad (3.7)$$

where $D = \rho[u^0(b^0)]/f[\bar{y}(b^0), u^0]$, $x \in m$.

4. Geometrical interpretation and qualitative conclusions [6].

(i) Function H can be expressed as $H = \sum_{i=1}^n \mu_i(\bar{y}, \bar{\psi}) \varphi_i(\omega)$, φ_i are continuous functions of x . The function H given in φ_i is referred to as a hyperplane of support to a vector-gradient, which defines direction of increase $\text{grad } H = \{\mu_1, \mu_2, \dots, \mu_n\}$. From this follows that H approaches maximum at one of the vertices of convex polyhedron Q , which represents a convex shell of the point set $\varphi_i(\omega_s)$, $\omega_s \in U$, $s = \overline{1, q}$. The remaining materials of the set U can be excluded from a further discussion.

(ii) The continuity of $\mu_i(x)$ implies that at any vertex of the polyhedron Q there is a hypercone K the interior of which may contain $\text{grad } H$ at slight variation in $x \in (x^*, x^{**}) \subseteq [0, b]$ and allow to satisfy the maximum condition. Thus, the optimal plate structure includes the finite number of layers of finite thickness.

(iii) Substitution of materials is expected to take place at the contact points x^* of the hypercone K and one of the polyhedron edges. It is to be noted here that the immediately adjacent materials may be only there which match the adjoining vertices of the polyhedron.

(iv) It can be shown that from the entire set of materials assumed in the vicinity of the rear surface the preference should be given to material with minimal density.

5. Numerical algorithm.

Numerical procedure requires insertion of some admissible control function $u(x) \in U$ and a small parameter χ which describes the set of measure zero. Computational algorithm involves the uniform mesh x_i having the mesh spacing χ . The values of \bar{y} and $\bar{\psi}$ are calculated at points $x_s + \chi/2$ and assumed constant for the segment $[x_s, x_{s+1}]$.

Solution includes the following steps:

(i) The system (2.4) is integrated and $\bar{y}(x)$ and b are defined at mesh nodes.

(ii) Boundary conditions for conjugate functions are prescribed at the point $x = b$ and the system (3.4) is solved.

(iii) A new value of u_s^* on the segment m_s is derived from condition $H(\dots, u_s^*) = \max_{\omega \in U} H(\dots, \omega)$; if $u_s^* = u(x_s + \chi/2)$ this step is repeated for $s = s+1$; otherwise, a new control function is assumed $u^* = u_s^*$ (if $x \in m_s$), $u(x)$ (if $x \notin m_s$) and calculation returns

to step (i).

The procedure of improving control function proceeds like this up to the terminal point on the right of the interval. The process is completed as soon as $u(x)$ remains constant at any s .

RESULTS

A set of materials contain annealed aluminum (a), aluminum alloy B-95 (b), annealed titanium (c), titanium alloy BT-6 (d), steel Cr.6 (e), steel 12X2H4A (f). Material properties are given in Table 1.

Table 1.

Material	Density ρ , g/sm ³	Young's modulus E , GPa	Yield stress σ_s , GPa	Poisson ratio, ν
a	2.8	70	0.06	0.33
b	2.8	70	0.45	0.32
c	4.5	110	0.08	0.30
d	4.5	120	0.83	0.32
e	7.8	200	0.21	0.28
f	7.8	200	0.11	0.30

For the case of dynamic penetration of tapered cylinder the optimal plate will consist of two materials (d)+(b). The relative front layer thickness of the optimal plate increases with the increase in a half-angle of the cylinder opening α and initial impact velocity v_0 .

Fig. 1 shows the decreased mass optimally \bar{F}_0 of homogeneous plates made up of (b), (d) and (f) -materials as compared to the optimal one. Disadvantage of heavy materials (d) and (f) decreases with the increase of v_0 , since their fraction in the optimal plate is growing high. At $v_0 < 600$ m/s the preference is given to a homogeneous material (b) rather than (d), while at $v_0 > 600$ m/s the preferred material is of d- type.

The results of present investigation agree qualitatively and quantitatively with data reported in [2]. If instead of material (f) one uses steel with the yield stress 1.5 GPa an optimal plate will consist of three layers. In case of a large choice of materials an optimal target structure will be multilayer. However, the main qualitative characteristic - a decrease of density and hardness with a distance from the upper to lower surfaces of the target - remains unchanged.

It is to be noted here, that the usefulness of a soft rear layer in a target has been already justified but only in context of fracture behavior of material. From mechanical point of view the optimality of target structure predicted in [2] and in present investigation implies its high resistance to penetration while preserving the same ductile type of cratering.

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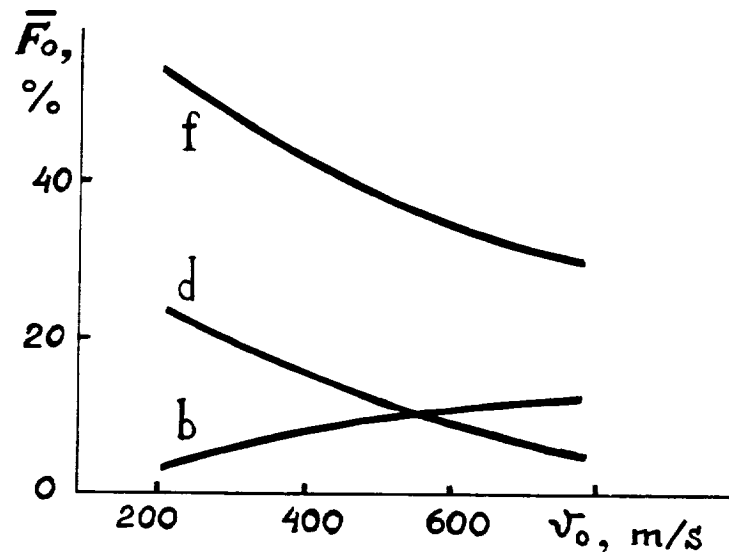


Fig. 1.